Gravitational Microlensing and the Structure of the Inner Milky Way

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ABSTRACT

We analyze the first-year MACHO collaboration observations of microlensing towards the Galactic center using a new direct likelihood technique that is sensitive to the distribution of the events on the sky. We consider the full set of 41 events, and calculate the direct likelihood against a simply-parameterized Galactic model consisting of either a gaussian or exponential bar and a double exponential disk. Optical depth maps are calculated taking into account the contribution of both disk lenses and sources. We show that based on the presently available data, a slope in the optical depth has been clearly detected (3σ) in Galactic latitude and that there are indications of a small slope in Galactic longitude. We discuss limits that can be set on the mass, angle and axis ratio of the Galactic bulge. We show that based on microlensing considerations alone, $M_{Bulge} > 1.5 \times 10^{10} M_{\odot}$ at the 90% confidence level and that the bulge inclination angle is less than 30° also at the 90% confidence level. The mostly likely bar mass is $M_{Bulge} = 3.5 \times 10^{10} M_{\odot}$. Such a high mass would imply a low MACHO fraction for the halo. We consider disk parameters and show that there are two degeneracies between the effects of a disk and those of a bar on the optical depths. Finally, we discuss how to break these degeneracies and consider various strategies for future microlensing observations.

1 Introduction

Although the Galaxy has been studied for a long time, determining its structure has proved to be an extremely difficult task. Our basic picture of the Milky Way as a spiral galaxy with a roughly exponentially falling disk, a central bulge and an extended halo has been settled for a considerable time[1]. Unfortunately, despite our wealth of detailed knowledge of the Galaxy, gaining a more precise knowledge of its global parameters, such as we have for external galaxies, is complicated by our position. Even such basic quantities as the scale length of the disk or the rotation curve of our own Galaxy are less well known than those of many external galaxies, due to the unfavorable geometry and intervening dust and gas[2, 3]. Because of our privileged position within it, the Milky Way provides us with a unique opportunity for studying questions such as how galaxies form or what their major constituents are. Ironically, many of the techniques we have for studying such questions are essentially measurements of light, while the basic questions have more to do with the mass and its distribution. The translation between these measurements and the information we would like is complicated by our fundamental lack of knowledge of the stellar mass function for small masses.

Gravitational microlensing searches are particularly exciting because, unlike other astrophysical observations, they can detect objects regardless of their luminosity. Within its mass range, a microlensing search is sensitive to the integrated density in massive compact objects of any type between the observer and source star. Originally intended for probing the baryonic component of the Galactic halo, gravitational microlensing has been increasingly recognized as a powerful new way to probe the structure of our entire Galaxy[4]. Although microlensing can also be used to probe the stellar mass function, we will concentrate on using it to directly constrain the mass density.

Over the past few years, microlensing has moved rapidly from a proposal to an established

fact. With its characteristically shaped light curve and achromaticity, there can be little doubt that microlensing has been observed in abundance towards the galactic bulge. Over one hundred events have now been observed by three collaborations, MACHO, DUO and OGLE, in the general direction of Baade's Window [5, 6, 7]. Small theoretically expected modifications to the light curve, such as effects from parallax and binary lenses have also been observed in some events, further confirming their interpretation as microlensing[8].

One of the most exciting of the recent microlensing results has been the observation of many more microlensing events in the direction of the Galactic bulge than had been predicted. One explanation for the higher than expected event rates is that the the Milky-Way is actually a barred spiral, with the bar oriented almost directly towards us[9]. A bar pointing at us would concentrate mass along our line of sight, increasing the number of microlensing events, but not leave an obvious signature of asymmetry in other observations. The suggestion that the galactic bulge is actually a bar is not new. As far back as 1964 de Vaucouleurs suggested a bar as a possible explanation for similarities between the gas dynamics seen towards the Galactic center and that seen in barred galaxies[10]. This idea, however, was not universally embraced. More recently, non-circular motions of the gas towards the Galactic center have been accounted for by a bar. A variety of other observations, such as star counts and luminosity studies, also indicate such a structure[11, 12, 13]. Consistent with this, although independently not compelling, are the infrared maps from the Diffuse Infra-Red Background Experiment (DIRBE) on the COBE satellite [14, 15]. Taken together, these more recent observations have led to a resurgence of the bar model. The high microlensing rate observed towards the Galactic center is not only consistent with this new picture, but can also be used as a probe to refine our knowledge of the Galactic structure and in particular the Galactic Bulge. A better understanding of the Galactic Bulge is important not only for its own sake, but also because of its interactions with the disk and halo. A bar may be intimately tied to the spiral density waves in our disk and as we discuss later, its mass plays

an important role in constraining our knowledge of the baryonic content of the halo[16].

Previous work comparing the results of microlensing searches to Galactic models has mostly focused on a single number: the microlensing optical depth in the direction of Baade's window[17]. However, in this region the optical depth is expected to be a rapidly varying function of latitude. The use of an average quantity, such as reported by the MACHO collaboration, limits our ability to match predictions from Galactic models with the data. Further, because a given optical depth is achievable in a variety of ways we cannot discriminate between the many possible models on the basis of this single number. It is only with using the gradient information, by a comparison of the distribution of the event locations to a map of the microlensing optical depth, that the various models can be sorted out. Thus we have developed a method for calculating the relative likelihood that a given map of the optical depth would produce the observed events. Using this likelihood technique we explore the constraints that the MACHO collaboration first year events impose on Galactic structure. We consider a class of models with bars based on the G2 and E2 models of Dwek et al. [15], and a simple double exponential disk. We calculate maps of the microlensing optical depth for each of these models and compare them to the observed events, calculating the likelihood as a function of the model parameters.

The paper is structured as follows: in the second section we briefly review the phenomenon of microlensing and the optical depth for microlensing, and develop the formalism we use in our likelihood analysis. We then discuss the models of Galactic structure we adopt for this paper and what constraints exist on the parameters for these models. In section three we examine the set of events and fields that we will use and discuss the observational efficiencies and sky coverage. We continue with a look at the structure of the data, determine whether the data support a gradient in the optical depth and produce a crude map of the optical depth. Section four contains the results of applying the likelihood formalism to the models mentioned earlier. We discuss the limits that can be placed on various parameters and

which parameters are correlated. The fifth section discusses future directions and examines strategies for maximizing what we learn from our microlensing investment. Finally in the last section we summarize the main results of this investigation.

2 Methods

2.1 Microlensing Optical Depth

When a massive object passes by the line of sight to a distant object, the object's image is distorted according to general relativity. Stunning confirmations of this have been seen in a variety of systems involving galaxy clusters and background quasars or galaxies where typical deflection angles are on the order of an arcsecond[18]. In microlensing the source is typically a distant star and the lens an intervening massive object. With the masses and distances much smaller the deflections are on the order of milliarcseconds, far too small to be resolved[19]. The distortion of the source shape is, however, not the only effect. The intervening mass also acts as a lens, concentrating the light of the source. The amplification is given by,

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \tag{1}$$

$$r_E = \sqrt{\frac{4GMd(D-d)}{c^2 D}} \tag{2}$$

where r_E is the Einstein ring radius, u is the dimensionless impact parameter, r/r_E, M is the mass of the lens, D is the distance to the source and d is the distance to the lens[20]. The amplification is more easily measured. When the dimensionless impact parameter, u, is less than one, we say that the lens is within the microlensing tube of the source. In this regime, the amplification is greater than about 1.34, corresponding roughly to experimental cutoffs.

What is probability that a given star is being microlensed? If this chance is small, as it is in all cases we consider, then it can be expressed as the typical number of lenses in the

microlensing tube. Thus we have the optical depth[21],

$$\tau = \int_0^L \frac{\rho}{M} \pi r_E^2 dl = \frac{4\pi G}{c^2} \int_0^D \rho(d) \frac{d(D-d)}{D} dd, \tag{3}$$

where ρ is the spatial mass density of lenses. Note that the dependence on the mass of the lens cancels out. This, together with the lack of a lens velocity dependence (which enters into calculations of the rate of microlensing events) is one of the great advantages of using the microlensing optical depth. It means we need not consider the mass spectrum of the lenses or their distribution in velocity space and can consider only their spatial mass density. We must, however, give up the possibility of using the distribution of event durations to learn about the stellar mass function or velocity distribution. In the formalism developed in the following we consider only the optical depth.

For a field with all sources at the same distance, such as the Large Magellanic Cloud or the Small Magellanic Cloud, Eq.(3) can be used directly to calculate the optical depth. If the source stars are at varying distances from the observer they will sample different optical depths. The observed depth is given by integrating over the source density along the line of sight:

$$\tau = \frac{4\pi G}{c^2} \frac{\int_0^\infty dD D^\alpha \rho_s(D) \int_0^D dx \rho_l(x) x(D-x)/D}{\int_0^\infty dD D^\alpha \rho_s(D)}, \tag{4}$$

where ρ_s is the mass density in source stars, ρ_l is the mass density in lenses, and α controls the source integration volume. Two factors enter into the determination of α. On the one hand, as the distance from the observer increases, the number of stars seen in a given solid angle will increase as the square of the distance. On the other hand, as the distance from the observer increases, fewer stars will be above the magnitude limit to be seen by the observer. For source stars on the main sequence these effects almost cancel out and $\alpha = 0$ is appropriate. For giants, such as the red clump giants used in the MACHO collaboration's analysis of a subset of their data, which can be seen throughout the bulge, $\alpha = 2$ is more reasonable. Since we use the full sample, which is composed of mainly main-sequence stars,

in our analysis we use $\alpha = 0$.

2.2 Likelihood

Although the average optical depth towards Baade's window is useful, it is not the entire story. In the standard technique one estimates the optical depth for a sample of events by summing up the total durations for stars that have been microlensed, $\hat{t}_i$, (weighted by the efficiency), and dividing by the total Exposure, E, (star years observed):[1]

$$\tau = \frac{\pi}{4E} \sum \frac{\hat{t}_i}{\epsilon_i} \tag{5}$$

This procedure inevitably loses information, because it averages over the events, ignoring their locations. For regions such as the MACHO fields, where there are strong variations of the optical depth, it is unclear how meaningful a procedure this is. The location at which one has measured the optical depth is undefined. This makes it difficult to compare ones' results to models of the optical depth. Additionally, it is difficult to reliably quantify the gradient information, even using latitude cuts, because of the uncertainty in the "distance" between the two sub-regions.

We would like to extract the gradient information and avoid the question of precisely "where" the optical depth is measured. For this we must deal with maps of the optical depth as a function of Galactic longitude and latitude and not simply average values. Central to this is an ability to quantify how well a given theoretical map of the optical depth compares to the observed events. Thus we construct a likelihood function sensitive not only to the number and durations of the events but also their positions. Constructing the required likelihood is not entirely trivial. It is instructive to look first at the case for maps of the microlensing rate. We then show how this is modified when dealing with the optical depth.

[1] Actually, the quantities extracted from the data, $\hat{t}_i$ measure the Einstein ring crossing time, not the time that the event is magnified above threshold. We adjust for this by using the average duration for an event with a given $\hat{t}$, $\pi\hat{t}/4$.

For any small patch of the sky, $dxdy$, the expected number of events in a time T is $T\Gamma dxdy$, where Γ is the rate per area. If n_i is the number of events actually observed in the ith sky-patch then by simple Poisson statistics the likelihood of the true rate being $\Gamma(x, y)$, given the observations, is

$$\prod_i e^{-T\Gamma dxdy}(T\Gamma dxdy)^{n_i}/n_i!. \tag{6}$$

Since for a sufficiently small patch size n_i will always be either 0 or 1 we arrive at

$$L = \exp\left(-T\int \Gamma dxdy\right) dx^N dy^N T^N \prod_{events} \Gamma(x, y), \tag{7}$$

where N is the total number of events. Dropping the $dx^N dy^N T^N$ factor, we have the relative likelihood for a model $\Gamma(x, y)$,

$$L = \exp\left(-T\int \Gamma dxdy\right) \prod_i^{events} \Gamma(x_i, y_i). \tag{8}$$

The case of optical depth is more subtle but parallel. We consider a patch $dxdydt$, where t is the time coordinate. Since the optical depth gives the probability that any given star will be lensed at a given time, the probability of observing n_i microlensing events in progress in this patch is

$$e^{-\sigma\tau dxdy}\frac{(\sigma\tau dxdy)^{n_i}}{n_i!}. \tag{9}$$

Here σ is the number density of source stars observed on the sky and τ is the optical depth in the patch. Note that this is independent of dt. For the entire region then, our likelihood is simply

$$L = \prod_{all\ cells}^{dxdydt} e^{-\sigma\tau dxdy}\frac{(\sigma\tau dxdy)^{n_i}}{n_i!}. \tag{10}$$

As before, for $dxdy$ small enough, $n_i \to 0, 1$ and so

$$L = \prod^{dt} \exp\left(-\int \sigma\tau dxdy\right) \prod_{events}^{dxdydt} \sigma\tau dxdy \tag{11}$$

where the second product is over all the cells containing events. Since the first term is independent of time the product over all intervals dt is easy. To do the second product we

note that for each event the average time spent in the microlensing tube for an event with the measured $\hat{t}$ is $\frac{\pi}{4}\hat{t}$. Hence we have

$$L = \exp\left(-\frac{T}{dt}\int \sigma\tau dxdy\right)\prod_{events}\left(\sigma\tau(x_i,y_i)dxdy\right)^{\frac{\pi\hat{t}_i}{4dt}}. \tag{12}$$

If all of the events have not been detected, that is the efficiency $\epsilon < 1$, then we will be missing terms in the product over events. To account for this we write

$$L = \exp\left(-\frac{T}{dt}\int \sigma\tau dxdy\right)\prod_{events}\left(\sigma\tau(x_i,y_i)dxdy\right)^{\frac{\pi\hat{t}_i}{4dt\epsilon_i}} \tag{13}$$

where ϵ_i is the efficiency for detecting events of length $\hat{t}_i$. Efficiencies for present experiments are typically of order 0.5. There is an immediate difficulty with this equation however: as $dt \to 0$ we have infinite exponents. This comes about because we are multiplying an infinite number of finite probabilities: one for each timeslice dt. What this procedure ignores is that there is a characteristic time interval over which microlensing in a cell $dxdy$ will be correlated. Thus we rescale the infinities with an ad hoc correlation constant t_0 which we will determine later. Thus we have finally,

$$L = \exp\left(-\frac{T}{t_0}\int \sigma\tau dxdy\right)\prod_{events}\left(\sigma\tau(x_i,y_i)\right)^{\frac{\pi\hat{t}_i}{4t_0\epsilon_i}} \tag{14}$$

where we have dropped the $dxdy$ in the product.

In the limiting case where $\tau = \tau_0$, a constant over the region of interest, we expect to recover the standard formula for optical depth. In this case our likelihood is

$$L = \exp\left(-\frac{T}{t_0}A\sigma\tau_0\right)\left(\sigma\tau_0\right)^{\frac{\pi}{4t_0}\sum_i\frac{\hat{t}_i}{\epsilon_i}} \tag{15}$$

where A is the area of the region and the sum is over the events observed. Setting the derivative with respect to τ_0 equal to zero we find

$$-\frac{T}{t_0}A\sigma L + \frac{\pi}{4t_0}\sum_i\frac{\hat{t}_i}{\epsilon_i}\frac{L}{\tau_0} = 0. \tag{16}$$

So the maximum is at

$$\tau_0 = \frac{\pi}{4TA\sigma}\sum_i \frac{\hat{t}_i}{\epsilon_i} = \frac{\pi}{4E}\sum_i \frac{\hat{t}_i}{\epsilon_i} \tag{17}$$

which is the standard formula. Looking at Eq. 14 we see that it can be rewritten as a scaled Poisson distribution

$$L = \exp\left(-\frac{T}{t_0}A\sigma\tau_0\right)\left(\frac{T}{t_0}A\sigma\tau_0\right)^{\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i}{\epsilon_i}}\left(\frac{T}{t_0}A\right)^{-\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i}{\epsilon_i}} \tag{18}$$

with maximum given above. This form allows us to fix t_0. We see that the distribution above has a width

$$\left(\frac{\delta\tau}{\tau}\right)^2 = \left(\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i}{\epsilon_i}\right)^{-1}. \tag{19}$$

We compare this to the expected uncertainty as calculated by Han & Gould [22]. We see that our

$$\frac{1}{\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i}{\epsilon_i}} = \frac{1}{N}\frac{\langle P^2\rangle}{\langle P\rangle^2} \tag{20}$$

in their notation. This works out to

$$t_0 = \frac{\pi}{4}\frac{\sum_i(\hat{t}_i/\epsilon_i)^2}{\sum_i \hat{t}_i/\epsilon_i} \tag{21}$$

which completely specifies our likelihood function.

The beauty of this likelihood function approach is in its flexibility. We can use it to directly compare the models to the data without first calculating an "observed" optical depth, or we can use it to explore what the data say about the optical depth. We will discuss later our construction of a primitive map of optical depth from the presently observed data. This approach also shines in the analysis of observations of a disparate collection of lines of sight, especially if some have low optical depth and produce no events. One simply uses a density function which is non-simply connected and the null data is taken into account automatically. Overlapping fields are also easy to handle in this formalism. Areas in which one has an overlap are simply counted as having twice the density.

We use our likelihood to rank the models we will discuss in the next section. It is important to note that we will be able to compute only relative likelihoods. The true structure of the Galaxy almost certainly does not fall exactly into the classes of models we consider. By considering a range of models that have passed a variety of other tests we hope to include at least some models that approximate reality.

2.3 Models

Our Galaxy can be described loosely as consisting of three parts: a disk, a dark halo, and a central bulge. In the following section, we describe the models we use in our calculations of optical depths. For each component, we indicate a range for its parameters that we feel is reasonable based on the literature. Because the values of these parameters are so uncertain we felt that it would indicate a false level of certainty to use a Gaussian prior for the parameters of our models. Accordingly, we have taken flat priors over the parameter ranges indicated. Since we expect the amount of microlensing due to objects in the halo to be negligible [21] and almost constant over the small range of directions examined, we do not model the halo.

The Galactic density enters the equations for the optical depth in two distinct ways: source and lens. One must therefore be very careful to keep the two roles separate. If both the bulge and the disk have the same lens to source ratio (or equivalently the same mass to light ratio), then the distinction becomes meaningless and can be dropped. It is not clear that this is the case. We handle the issue in the following manner. The visible content of the disk, that which could be seen as sources, is relatively well known. On the other hand, the ratio of bulge to disk source stars in the MACHO fields has been estimated by the MACHO collaboration as around 20% based on the fraction of the 2.2 micron flux contributed by the disk[5]. We therefore model the disk with a source disk and a lens disk, and adjust the bulge source fraction to give approximately an 80% contribution along our line of sight.

2.3.1 Bulge

Models of the Galactic bulge are very uncertain. The difficulties are both practical (most fields towards the bulge suffer extremely high extinction) and theoretical (it is extremely difficult to invert gas and stellar dynamics to obtain potentials). Nevertheless, a number of models have emerged as standards[15]. These models have widely ranging functional forms. We consider two representative forms, Dwek et al's G2 and E2. Their densities can be expressed

$$\rho_{G2} = 1.2172 \frac{M_{BAR}}{8\pi x_0 y_0 z_0} e^{-r_s^2/2} \tag{22}$$

$$\rho_{E2} = \frac{M_{BAR}}{8\pi x_0 y_0 z_0} e^{-r} \tag{23}$$

$$r_s = \left\{ \left[\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 \right]^2 + \left(\frac{z}{z_0}\right)^4 \right\}^{1/4}$$

$$r = \left\{ \left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 + \left(\frac{z}{z_0}\right)^2 \right\}^{1/2}$$

with the major axis inclined towards us at an angle θ_B, which we will take to be between $0°$ and $45°$, with the near side in the first Galactic quadrant. The G2 model is the best fit to the DIRBE infrared maps [15], while the E2 is favored by an analysis of the distribution of red clump giants in the OGLE fields[23]. At the location of the MACHO fields we will be analyzing, G2 models tend to have high optical depths while E2 models produce optical depth considerably less efficiently [24]. By considering both models with a wide range of parameters we hope to cover a large portion of possible bulge models.

Estimates of bulge masses cover a wide range. More recently, however, estimates have fallen approximately in the range $(2.0 - 3.0) \times 10^{10} M_\odot$[24]. To be conservative we consider the range $1.0 - 4.0 \times 10^{10} M_\odot$. The bulge scale factors are less well known. The best known of these are the two vertical scale heights. We follow the results of Stanek et al.[23] and fix

these at 0.43 kpc for the G2 models and 0.25 kpc for the E2 models. We consider the ranges $0.3 - 2.7$ kpc (G2) and $0.2 - 1.8$ kpc (E2) for both x_0 and y_0. Following the review paper by Kerr & Lynden-Bell [25] we fix the distance to the bulge to be the IAU recommended value of 8.5kpc.

2.3.2 Disk

As discussed above the structure of the luminous component of the disk is relatively better known than a possible dark component [26]. Thus we consider two separate disks. The first is a thin luminous double exponential disk,

$$\rho_{lum} = \frac{\Sigma_{lum}}{2r_z} \exp\left(-\frac{z}{r_z}\right) \exp\left(\frac{r_0 - r}{r_d}\right), \tag{24}$$

composed of luminous stars that can serve as sources. The parameters for this disk are fixed: $\Sigma_{lum} = 15 M_\odot \,\mathrm{pc}^{-2}$, $r_z = 0.3\,\mathrm{kpc}$, and $r_d = 3.5\,\mathrm{kpc}$.[27, 26] Not all stars in the disk are bright enough to been seen, however, and in fact there is evidence that the disk contains considerable mass beyond that which is visible, perhaps distributed somewhat differently from the luminous mass [28, 29, 30]. Hence we consider also a second double exponential disk,

$$\rho = \frac{\Sigma_0}{2r_z} \exp\left(-\frac{z}{r_z}\right) \exp\left(\frac{r_0 - r}{r_d}\right), \tag{25}$$

whose parameters are not fixed. This component is allowed to serve as lenses. Analysis of the vertical velocity distributions of stars in the vicinity of the sun, gives 2σ upper limits on the total surface density within 1 kpc of the disk plane of at most $85 M_\odot \,\mathrm{pc}^{-2}$[28, 29, 30]. Of this about $30 M_\odot \,\mathrm{pc}^{-2}$ is in the form of bright stars and gas, while perhaps $10 M_\odot \,\mathrm{pc}^{-2}$ is in the form of M dwarfs which could serve as lenses. About $10 M_\odot \,\mathrm{pc}^{-2}$ is contributed by the halo, more in flattened models. This leaves a maximum of $35 M_\odot \,\mathrm{pc}^{-2}$ for a possible dark disk. Adding in the M dwarf lenses, the surface density of lenses should be in the range $10 - 45 M_\odot \,\mathrm{pc}^{-2}$. To be conservative we choose the range $10 - 55 M_\odot \,\mathrm{pc}^{-2}$. The scale

height for the dark component of the disk is unknown. We therefore consider the range $0.2-1.5\,\mathrm{kpc}$, which corresponds to populations tighter than the visible stars at the low end, to a significantly heated population at the high end. We fix the disk scale length at a value of 3.5kpc[3].

3 Data

3.1 Events & Fields

The basic requirement for microlensing searches is a very large sample of distant stars. For studies of the halo the obvious choices of fields are towards the Large Magellanic Cloud and the Small Magellanic Cloud. Searches towards the bulge are complicated by the extinction from intervening dust obscuring the Galactic Center. In the red and blue bands used by the MACHO collaboration, there are only a relatively few fields with a large enough density of stars. One of these "holes in the dust" is Baade's Window at Galactic longitude (l) and latitude (b) (1°,-4°), towards which the first-year observations are clustered. We show in Fig. 1 a plot of the 24 fields reported on by the MACHO collaboration. For reference, also included are some of the OGLE fields, which are also of relatively high density. The MACHO collaboration observed 12.6 million stars for a period of 190 days during the 1993 season in these 24 fields[5]. The 41 events that pass their criteria for microlensing are plotted on Fig. 1 as dots with radii proportional to their duration. The location and duration of each event can be found in Alcock et al. Table 1[5]. In addition to the location and duration of the events we also need a variety of other numbers. Information on the location, size and orientation of the 24 fields can be found at the MACHO website, http://wwwmacho.anu.edu.au. The

Figure 1: $20° \times 20°$ region centered at the galactic center. Light boxes are the 24 MACHO 1st year fields. Smaller heavy boxes are OGLE fields. The 41 events we analyse are shown as dots with radii proportional to the event duration.

two quantities that pose the greatest problems are the observational efficiencies and the density of the stars in each field. As of this writing, the MACHO collaboration has not completed its analysis of the full blending efficiencies for the bulge fields. As a reasonable approximation, they suggest using their sampling efficiencies with a 0.75 factor correction. Accordingly, we following this prescription and use their standard cut sampling efficiencies from Fig. 5 of Alcock et al[5]. Since the fields under consideration are similar and all are crowding limited, we assume that the efficiencies are uniform across the sample. The reader is warned however, that this is a major source of uncertainty. It is possible that the efficiency is not only a function of duration but also of position [22] which would introduce spurious spatial structure into the microlensing distribution.

The final quantity we need for our analysis is the density of observed sources. In view of the fact that the fields are crowding limited and in the absence of better data, we assume a uniform density of source stars across the fields. Accounting for overlap we get $\sigma = 1.06 \times 10^6 \mathrm{deg}^{-2}$. We note an encouraging point. The efficiencies are likely to get better as the fields get less crowded since blending effects are smaller. On the other hand, less crowded fields mean a smaller source density. Hence the errors due to our assumptions of constant efficiencies and source densities should be in opposite directions and are likely to at least partly cancel.

3.2 Structure of the Data

Before attempting to extract information about the structure of the Galaxy from the data, we first look at what we can learn about the structure of the data itself. We apply our likelihood formalism to the very simplest model of the optical depth possible:

$$\tau = \tau_0, \tag{26}$$

a flat optical depth over the entire set of fields. We obtain the result $\tau_0 = 1.93 \pm 0.39 \times 10^{-6}$ where we quote 68% confidence limits. The MACHO collaboration's reported value of $2.4 \pm 0.5 \times 10^{-6}$ for the same events includes a "correction" factor of 1/0.8 introduced to adjust for contamination by source stars in the disk. Undoing this "correction", we obtain $1.9 \pm 0.4 \times 10^{-6}$, in agreement with our result. Note that our errors are from our analytic calculations and not the result of Monte Carlo simulations.

3.3 Gradients

We next apply our formalism to a slightly more complicated set of models: those with an optical depth gradient in the b and l directions. We consider models with the form,

$$\tau = \tau_0 + \frac{d\tau}{db}(b - b_0) + \frac{d\tau}{dl}(l - l_0), \tag{27}$$

where $b_0 = -4$ and $l_0 = 2$. Fig .2 shows contours of the likelihood, marginalized over τ_0, in the $\frac{d\tau}{db} - \frac{d\tau}{dl}$ plane. A gradient in the optical depth in latitude is clearly indicated. The case for a slope in longitude is less clear-cut. Marginalizing over the remaining parameter in each case we obtain $\frac{d\tau}{db} = 1.12 \pm 0.37 \times 10^{-6}$/deg and $\frac{d\tau}{dl} = -1.71 \pm 1.19 \times 10^{-7}$/deg, again with 68% confidence limits.

Our latitude gradient can be compared with an estimate based on the MACHO clump giant optical depths reported for fields above and below $b = -3.5°$[5]. Scaling the calculated slope, $s = (6.32 - 1.57) \times 10^{-6}/1.38°$, to the full sample and undoing their 1/0.80 disk correction, we obtain an estimate of $\frac{d\tau}{db} = 1.7 \times 10^{-6}$/deg with likely errors of at least 1.0×10^{-6}/deg. This is fully consistent with our results.

3.4 Maps

Just how much information about the structure of the event distribution can we extract? Our likelihood method lends itself nicely to the construction of the model independent "most

likely" map of the microlensing optical depth. Consider a general function $\tau(b, l)$ giving the optical depth. We would like to find the positive definite function $\tau(b, l)$ which maximizes the likelihood or equivalently minimize the negative log likelihood,

$$LL = \int\int \left[\frac{T}{t_0}\tau(b,l)\sigma(b,l) - Q(b,l)\log\left(\tau(b,l)\right)\right] dbdl \tag{28}$$

where

$$Q = \frac{\pi}{4t_0}\sum_{events}\hat{t}_i\delta(l-l_i, b-b_i). \tag{29}$$

Taken alone, however, this condition is insufficient. The solutions to this equation turn out to be delta functions at the event locations, a clearly unphysical situation. What is missing is that the optical depth should be a smooth function of position on the sky. Thus we must add a smoothing term to the log likelihood to be minimized. Our smoothing term must discourage structure unmotivated by the data, yet at the same time not penalize legitimate gradients such as we have seen in the data. We choose to minimize the extrinsic curvature, given by

$$K = \frac{d^2\tau}{db^2} + \frac{d^2\tau}{dl^2} + 2\frac{d^2\tau}{dldb}. \tag{30}$$

Thus we require a minimum of

$$LL = \int\int \left[\frac{T}{t_0}\tau(b,l)\sigma(b,l) - Q(b,l)\log\left(\tau(b,l)\right) + \lambda K^2\right] dbdl \tag{31}$$

where λ controls how much smoothing we require.

We solve for τ by an iterative scheme, starting with a flat $\tau(l, b)$. Setting λ to provide a reasonable smoothness we produce the map shown in Fig. 3. Our generated map contains few surprises. We note a pronounced tilt in galactic latitude and a small one in galactic longitude just as we saw in the earlier sections. A slight bending of the contours to wrap around the Galactic center is also present. It is important to remember that although the generated map is smooth and does not appear "noisy", this is an artifact of the way it is created: the smoothing term ensures that the resulting map is fairly smooth. The significance of the

present map is low, due to the small number of events. A considerably larger data set would be needed before the map could be used to yield detailed information.

4 Results

With the results from our look at the data in mind, we now consider the more realistic models of the Galaxy discussed above. For each type of bulge, G2 and E2, we calculate the likelihood as a function of the various Galactic parameters with the stated ranges. Since the functional form of the bulge is so poorly constrained, we resist the temptation to make a direct comparison between the two bulge models. Given the number of parameters in our models and the present number of events, any such comparison would be of marginal significance. Instead, we focus on the parameters which have meaning independent of the functional form such as the mass of the bar, M_B, the inclination angle of the bar, θ_B and the bar axis ratio, $r = \frac{x_0}{y_0}$. In this section we will discuss the limits we can put on bulges of only these functional forms.

4.1 Bar

Due to the large number of parameters in our full model and limits on computational power, in our exploration of the implication of the microlensing events for bulge parameters we fix the disk parameters to reasonable values: $\Sigma_0 = 30.0, r_z = 0.3\,\mathrm{kpc}$, and $r_d = 3.5\,\mathrm{kpc}$, while varying M_B, θ_B, x_0 and y_0. The bulge quantities we are most interested in, M_B, θ_B and r, should be most strongly influenced by the magnitude and longitudinal gradient of the observed optical depth. Of these, only the magnitude of the optical depth is sensitive to the disk parameters. We expect that as we increase the disk surface density, the inferred mass will decrease. We have checked our results and find that this effect amounts to a less than $0.3 \times 10^{10} M_\odot$ shift even when we increase the disk to $55 M_\odot\,\mathrm{pc}^{-2}$. Our limits on other bulge parameters are unaffected.

4.1.1 Bar Mass and Orientation

Our major results concerning the bar mass and orientation are summarized in Figs. 4&5. These figures show contours of likelihood as a function of the mass of the bar and orientation angle away from our line of sight. The two horizontal scale lengths, x_0 and y_0 were marginalized. Fig. 4 shows results for G2 models, while Fig. 5 presents E2 models. We discuss the G2 case first.

One feature of Fig. 4 is immediately apparent: mass can be traded off for angle. A ridge in the likelihood lies on the line $M_{BAR}(10^{10}M_\odot) - 0.11\theta_B(\text{deg}) = 1.4$. There are limits to this trade-off, however. If the mass of the bar is much beyond $4.0 \times 10^{10}M_\odot$, too high an optical depth will be produced to fit the data even if the angle is increased dramatically. There is also a sharp cutoff when the mass drops to below about 1.7. At such low masses, decreasing the angle no longer helps but rather hurts since the maximum optical depth is at a non-zero angle.[24] The situation is much the same for the E2 models (Fig. 5), except shifted by about $1.1 \times 10^{10}M_\odot$ in bulge mass. The E2 models drop off too rapidly to be efficient at producing microlensing optical depth even when optimally aligned, and hence need considerably higher bar masses[24]. Below a mass of about $2.0 \times 10^{10}M_\odot$ it becomes difficult to produce the high optical depths required by the data. We show in Fig. 6 the likelihood for the bar mass now marginalized over the bar orientation as well. The 90% confidence limit is at $1.75 \times 10^{10}M_\odot$ for the G2 bulge and $2.6 \times 10^{10}M_\odot$ for the E2 bulge. Taking into account the uncertainty in the disk normalization, we arrive at lower bounds on the bulge mass of $1.5 \times 10^{10}M_\odot$ (G2) and $2.3 \times 10^{10}M_\odot$ (E2). The most likely values are around $3.5 \times 10^{10}M_\odot$ for G2 models and beyond $4.0 \times 10^{10}M_\odot$ for E2 models.

In Fig. 7 we plot the marginalized likelihood versus bulge inclination angle. Low inclination angles are clearly favored. The likelihood has dropped off strongly by 30° and 20° for the G2 and E2 models respectively. Beyond these angles, even bar masses as high as

$4.0 \times 10^{10} M_{\odot}$ cannot produce enough microlensing to be compatible with the experimental results. The 90% confidence limits are 30° (G2) and 21° (E2).

Our results, using only a single year of microlensing data, fit very nicely with attempts to constraint the bulge mass and orientation angle by other means. Analysis of the stellar motions in the bulge gives a range of mass estimates from slightly below $2.0 \times 10^{10} M_{\odot}$ to almost $3.0 \times 10^{10} M_{\odot}$.[31, 32, 33] Several authors have derived bulge masses around $2.2 \times 10^{10} M_{\odot}$ using a variety of methods based on modeling of the gas content of the bulge.[11, 34] A simple argument by Zhao et al. gives this as an upper limit[24]. Our limit on the bulge mass is consistent with the reported values for the bulge mass.

The OGLE collaboration has reported an analysis of the distribution of the bulge red clump giants within their fields shown in Fig. 1. They report a bulge orientation of between 20 and 30° almost independent of the bulge model.[23] Dwek et al. analyze the DIRBE maps of the infrared emmision from the bulge and conclude that the orientation angle lies in the range $10 - 40°$. Our range of $0 - 30°$ is also consistent with the value $16 \pm 2°$ suggested by Binney et al.'s analysis of gas dynamics[11].

Perhaps more important than our limits on M_B and θ_B separately, are the full contours of likelihood in the M_B - θ_B plane showing the correlation between high bulge mass and high orientation angle. As we discuss later, increases in the number of events at Baade's window do little to break the M_B - θ_B degeneracy, but make the ridge considerably narrower. It requires more information to uniquely pick out a bulge mass. An analysis using the tensor virial theorem for the bulge by Blum [33] gives exactly the opposite degeneracy: high mass is correlated with low angle. Thus the results of microlensing and dynamics arguments are complementary and may be able to break the mass-angle degeneracy for either alone.

4.1.2 Bar Axis Ratios

The other bulge quantity we look at is the bar axis ratio $r = \frac{x_0}{y_0}$. Although we do not explicitly have the axis ratio as an input quantity for our bulge models, microlensing puts limits on the ratio of the scale lengths. In Figs. 8&9 we have marginalized θ_B and the x_0-y_0 pair subject to the constraint $r = \frac{x_0}{y_0}$. One can immediately see a trade-off between the axis ratio and the mass. A higher axis ratio concentrates the mass where it will do the most microlensing. Hence, in conjunction with a low orientation angle, this allows a lower mass. Since a bar-like configuration seems to be favored by the most recent data on the bulge it is interesting to note that our results do not completely rule out axisymmetric models. The preferred axisymmetric models have very high masses. On the other hand, measurements of velocity dispersions in the bulge give low constraints on masses of axisymmetric models. The strongest microlensing evidence against axisymmetric models comes from a consideration of the more limited bulge giant subsample of the events, which probe the optical depth for sources distributed throughout the bar. An axisymmetric model can not produce enough microlensing to account for the high optical depth implied by these events[16]. Our most likely values, r=3.5 (G2) and r=2.5 (E2), are consistent with the axis ratios determined by other means. Dwek et al. give a range $2.5 - 5.0$ for their models[15]. Stanek et al. obtain a value in the range $2.0 - 2.5$ again independent of model choice[23]. The distribution of bulge Mira variables gives an axis ratio of 3.9.[12].

4.2 Disk-Bulge Discrimination

Since we hope to discriminate between the bulge and disk on the basis of the latitude gradient information, the parameters M_B, θ_B, r_z, and Σ_0 are most relevant. Thus we vary these quantities while holding the two horizontal bulge scale lengths constant at $x_0 = 1.58\,\mathrm{kpc}$ and $y_0 = 0.62\,\mathrm{kpc}$ for G2 models. The results for the E2 models are similar to those for the G2 models, except for a shift in the bulge mass as noted above. Since we are interested only in

the disk parameters we discuss only the results for the G2 models.

4.2.1 Surface Density

Fig. 10 shows the marginalized likelihood as a function of the surface density of the disk versus mass of the bulge. It is immediately clear that we cannot uniquely fix the surface density of the disk. Rather, as was the case with the orientation angle of the bulge, Σ_0 shows a linear degeneracy with M_{BAR}, with the ridge of the likelihood at $M_B(10^{10}M_\odot) + 0.0088\Sigma_0(M_\odot/pc^2) = 2.88$. A heavy disk can add as much as 0.7×10^{-6} to the optical depth, allowing the bar to be less massive. The widening of the likelihood contours towards the top shows a slight tendency towards a more massive disk. This tendency, although not significant, is due to the slope of the optical depth in latitude.

4.2.2 Scale Height

We show in Fig. 11 the likelihood as a function of the scale height of the disk and the bar mass. The scale height is only very weakly correlated with the bar mass and is basically unconstrained. There is a spreading of the likelihood contours for low scale heights possibly favoring scale heights below about 0.6 kpc. At low scale lengths the gradient in optical depth of the disk contribution is high. As the scale length increases, the disk gradient decreases. Hence low values of r_z are favored, with high values, where the gradient is low, suppressed.

4.2.3 Disk Bulge Degeneracy

Our results for disk parameters are something of a disappointment. With the present data we can say virtually nothing about the disk structure. Part of the problem is the trade-off that occurs between M_B and Σ_0, keeping the optical depth constant and producing the ridge in the M_B versus Σ_0 likelihood plot. We had hoped, however, that gradient information would allow us to break the degeneracy between M_B and Σ_0. The reason that this does not happen is not that the area that the MACHO fields span is too small to show strong structure

with these few events; slopes in b and l are indicated. The problem lies with the position of Baade's window and the details of the Galactic models. Firstly, because the longitudes of the bulk of the MACHO fields are low, the longitude slope expected in this region is small for virtually any model. This makes the slope in longitude a poor diagnostic. Secondly, at the location of Baade's window the M_B - Σ_0 trade-off also keeps the latitude gradient fairly constant over a large range. We show this in Fig. 12, where the latitude gradient is shown as a function of Σ_0, with the optical depth kept constant by varying the M_B. The different curves show various values of the scale height of the disk. Since the b slope in the region of Baade's window is constant for a wide range of models it is very difficult to discriminate among them. This is why we see only hints of structure in our likelihood plots.

5 Future Directions

Over the next year, two collaborations, OGLE and EROS, will be moving to dedicated telescopes and fully automated analysis systems. Together with the currently running MACHO system, these groups have the potential for producing many times the data we have used here. How will such a wealth of data effect our results? To explore this question we have synthesized 4 years of observations and rerun our analysis. Our synthetic observations were constructed assuming a G2 model with $M_B = 3.0 \times 10^{10} M_\odot, x_0 = 1.58\,\mathrm{kpc}, y_0 = 0.62\,\mathrm{kpc}, \theta_B = 15°, \Sigma_0 = 30.0 M_\odot\,\mathrm{pc}^{-2}, r_z = 0.3/kpc$. The total number of events expected was calculated as

$$N_{exp} = \frac{T}{\langle t \rangle} \int \sigma\tau(l,b) dl db \qquad \mathrm{w}/\langle t \rangle = \frac{\pi}{4} \sum_i \frac{\hat{t}_i}{\epsilon_i} / \sum_i \frac{1}{\epsilon_i}, \tag{32}$$

where the average was done over the present data set and the integral is over the MACHO first-year fields. From this expected number, the actual number was picked assuming Poisson statistics. Since the rate is proportional to the optical depth (assuming constant average duration), the events were laid down randomly according to the optical depth. A duration

was picked out of the efficiency weighted set of observed durations. The proposed event was then "observed" or not based on the efficiency for that duration. The final set of "observed" events was run through our analysis. The results are shown in Fig. 13. We note a number of features of the results. First is that the confidence regions have tightened somewhat as expected. Second, the basic degeneracy between M_B and θ_B is uneffected. Although θ_B now seems to be quite well constrained, M_B still varies over a wide range. As we discussed earlier, part of the problem is in the distribution of our present lines of sight. Baade's window is a poor location for determining longitude slope, and the latitude slope is correlated with the optical depth, making it less useful as a diagnostic tool. If we wish to determine the bulge parameters solely from microlensing, simply collecting more data in the same fields will not easily break the degeneracies between parameters. We must look to expanding our range of fields.

So, what is the best strategy to use? Microlensing searches are costly and very time-consuming: we would like to find a strategy that maximizes the scientific return. We attempt to address this issue by analyzing a set of very stylized strategies that will allow us some insight into real searches.

Let us assume that we observe four identical fields centered at $(1.0, -3.0)$, $(1.0+\Delta l, -3.0)$, $(1.0, -3.0 - \Delta b)$ and $(1.0 + \Delta l, -3.0 - \Delta b)$. Further, we take the limit as the size of the fields goes to zero but the exposure for each field, E, stays constant. In this way the only gradient information will come from the field separation, and not from the distribution of events within the fields. Each choice of Δl, Δb will be a distinct strategy. Let $\hat{t}_i^j$ represent the durations of the events observed in the j-th field. Then the likelihood for a given model, $\tau(l, b)$ is

$$L = \prod_j e^{-\frac{E}{t_0}(\tau_j)} (\sigma\tau_j)^{\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i^j}{\epsilon_i}} \tag{33}$$

where τ_j is the predicted optical depth at the j-th field. Let $\tau^0(l, b)$ represent the optical

depth of the underlying model. Then on average $\frac{\pi}{4t_0}\sum_i \frac{\hat{t}_i^j}{\epsilon_i} = \frac{E}{t_0}\tau_j^0$ and we will have

$$L = \prod_j e^{-\frac{E}{t_0}(\tau_j)}(\sigma\tau_j)^{\frac{E}{t_0}\tau_j^0}. \tag{34}$$

Using this likelihood we can now determine how well a given strategy can recover our underlying model. Figs. 14,15&16 show the magnitude of the 68% confidence intervals in various quantities as a function of the longitude and latitude separation of the fields assuming twice the exposure of present experiments. We used the same model as in the previous section. The first thing one notices is that the varying strategies don't make as much difference as might be hoped. As the separation of fields is increased, our lever arm for making determinations of the quantities increases. However, at the same time the outer fields are in regions with low τ and hence few events and poor statistics. These two effects tend to cancel out, making dramatic improvements difficult. Nevertheless, what can we learn from these graphs? First of all, the present data correspond to roughly $\Delta l = 3.0$, $\Delta b = 2.0$. This is very close to the worst possible region for determination of all three parameters.

If we wish to improve our determination of M_B the results suggest a relatively large Δb and a smallish Δl. For Σ_0, on the other hand, we should have a high Δl and Δb is irrelevant. Finally, for θ_B, a moderate Δl is required and again Δb is mostly unimportant. A single search strategy to fix the parameters would need to probe the entire range of scales in longitude. Coverage in latitude appears to be less important; only the large scales need to be probed. The best strategy would seem to be one which includes many fields scattered over the entire bulge instead of concentrated in one region. Although the optical depth at any given location would be less well defined, better limits on the global parameters would be obtained.

6 Conclusions

In this paper we have developed a novel likelihood technique for the analysis of the microlensing data towards the bulge. We construct a likelihood that is sensitive to the spatial distribution of the events. Our technique is both more flexible than calculations that have been done before, and allows for a direct comparison of the data to models of the mass distribution for the Galaxy. It is particularly good for dealing with data from more than one line of sight, field overlaps and variations in the density of stars observed. Its sensitivity to the position of events makes it ideal for determining gradients in the optical depth. Applying this technique to the first year MACHO data we have confirmed the strong slope in latitude found by the MACHO collaboration, and found hints of one in longitude. We have for the first time, given quantitative measure of these slopes, $\frac{d\tau}{dl} = -1.71 \pm 1.19 \times 10^{-7}$/deg and $\frac{d\tau}{db} = 1.12 \pm 0.37 \times 10^{-6}$/deg. We also apply our analysis to constructing a crude "most likely" map of the microlensing optical depth over the observed region.

We confront a set of Galactic models consisting of a bulge with either G2 or E2 functional form and an exponential disk, with the data. With only one season of microlensing data, we can already set meaningful limits on various bulge parameters. We find that $M_B > 1.5(2.0) \times 10^{10} M_\odot$, for the G2 (E2) based models. Most likely values for the bulge mass are much higher: $M_B = 3.5(> 4.0) \times 10^{10} M_\odot$. Previous work has shown that such high bulge masses imply low halo MACHO fractions[16]. A massive bulge puts tight constraints on the contribution of the disk to the rotation curve at small radii. A small disk, however, leaves more room for the halo in the outer rotation curve, implying a massive halo. Since microlensing results towards the LMC fix the MACHO content of the halo, a massive halo implies a smaller MACHO fraction.

We also constrain the inclination angle of the bulge finding that $\theta_B < 30°(21°)$, consistent with other measurements. Our most likely values for the axis ratio of the bulge,

$r = \frac{x_0}{y_0} = 3.5(2.5)$, are consistent with determinations by other methods. We note that axisymmetric bulge models are not entirely ruled out with the full sample of events. Such models, however, typically need very high ($\approx 4.0 \times 10^{10} M_\odot$) bulge masses. Such high bulge masses are unlikely for axisymmetric bulge models[35]. No limits could be set on the disk component due to a degeneracy in the latitude slope of the optical depth between the bulge and disk contributions. We discuss what can be expected with an increase in the number of seasons of data.

Finally, we have attempted to quantitatively discuss various strategies for microlensing searches and conclude that a strategy observing many fields well scattered in longitude offers the best return in determining bulge and disk parameters. The distribution of fields in latitude is less important. Despite the difficulties, this is perhaps more important than simply getting the optical depth more accurately at one location as it will allow a better determination of the relative contributions of the bar and the disk. Ideally, the optical depth can be mapped over a wide range in both latitude and longitude, yielding detailed information about the mass distribution in the inner Galaxy. The field of microlensing promises to be an eventful one for the foreseeable future!

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Figure Captions

Figure 1: $20° \times 20°$ region centered at the galactic center. Light boxes are the 24 MACHO 1st year fields. Smaller heavy boxes are OGLE fields. The 41 events we analyse are shown as dots with radii proportional to the event duration.

Figure 2: Likelihood contours for l and b gradients in the optical depth. Solid lines are the 68% confidence contours. Dotted lines denote 95% confidence. Dashed lines denote 99% confidence. Also included, to guide the eye are long dashed lines for 38% confidence.

Figure 3: "Most likely" map of the optical depth based on the first year events. The solid lines show contours of $4.0, 3.0, 2.0, 1.0, \text{and} 0.5 \times 10^{-6}$ from the top down.

Figure 4: Contours of likelihood in the $\theta_B - M_B$ plane for G2 models with x_0 and y_0 marginalized. Disk values where held fixed at $\Sigma_0 = 30 M_\odot pc^-2$ and $r_z = 0.3\,\text{kpc}$. Contours are as in Fig. 2.

Figure 5: Same as Fig. 4. but for E2 models.

Figure 6: Likelihoods from Figs. 4&5 now with θ_B marginalized. Solid line for G2 models, dashed line for E2 models.

Figure 7: Same as Fig. 6, but with M_B marginalized.

Figure 8: Contours of likelihood in the M_B - r(axis ratio) plane. θ_B and one degree of freedom from x_0, y_0 have been marginalized. As in Fig. 4, disk parameters have been fixed. Contours are as in Fig. 2.

Figure 9: Same as Fig. 8 but for E2 models.

Figure 10: Contours of likelihood in the M_B - Σ_0 plane. θ_B and r_z have been marginalized. Bulge parameters x_0 and y_0 have been held fixed at $x_0 = 1.58\,\mathrm{kpc}$ and $y_0 = 0.62\,\mathrm{kpc}$. Contours are as in Fig. 2.

Figure 11: Contours of likelihood in the M_B - r_z plane. θ_B and Σ_0 have been marginalized. Bulge parameters x_0 and y_0 have been held fixed at $x_0 = 1.58\,\mathrm{kpc}$ and $y_0 = 0.62\,\mathrm{kpc}$. Contours are as in Fig. 2.

Figure 12: Slope in latitude for the optical depth as a function of Σ_0, the disk surface density. The total optical depth has been held constant by varying M_B as Σ_0 varies. The lines represent: solid ($r_z = 0.3\,\mathrm{kpc}$), dashed ($r_z = 0.5\,\mathrm{kpc}$), long dashed ($r_z = 1.0\,\mathrm{kpc}$), and dot-dashed ($r_z = 1.5\,\mathrm{kpc}$). Total variation across our range of disk surface densities is less than 10%.

Figure 13: Likelihood in the M_B - θ_B plane for a simulated 4 seasons of data based on a G2 model with $M_B = 3.0 \times 10^{10} M_\odot, x_0 = 1.58\,\mathrm{kpc}, y_0 = 0.62\,\mathrm{kpc}, \theta_B = 15°, \Sigma_0 = 30.0 M_\odot\,\mathrm{pc}^{-2}, r_z = 0.3/kpc$. Contours same as Fig. 2.

Figure 14: Contours of the size of the 68% confidence regions in the determination of M_B. Values are calculated as a function of the latitude and longitude spacing of the observed

fields. Solid: 1.46, Dotted: 1.53, Dashed: 1.61, Long Dash: $1.69 \times 10^{10} M_\odot$

Figure 15: Same as Fig. 14 but for θ_B. Solid: 16°, Dotted: 19°, Dashed: 23°, Long Dash: 26°

Figure 16: Same as Fig. 14, but for Σ_0. Solid: 29, Dotted: 35, Dashed: 42, Long Dash: $48 M_\odot\,\mathrm{pc}^{-2}$.